تم الرفع بواسطة
م.معن ابو عيسى

Calculus 2
Second

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[redacted] Palestine Technical University
Department of Applied Mathematics.
Calculus II.

Student # معن سلام محمود قعدان **Second Exam** May, 19th, 2010.

Student name بالعربيه: Time: 60 min.

احمد/ثلاثاء/خميس
1200 - 1:00

Instructors: Dr. Khalida Nazzal. Mrs. Kifaya Ayash.
Mr. Jasem Badran. Mr. Saleh abou Serees.

Problem 1. (40 pts.) Circle the correct answer.

1) If $\cosh x + \sinh x = 1$, *then* $x =$

a) 3
b) 2
c) 1
d) 0.

$$\frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2} = 1$$

$$e^x + e^{-x} + e^x - e^{-x} = \frac{1}{2}$$

$$e^x = 2 \qquad \frac{2e^x}{2} = 1$$

$$\frac{e^x}{2} + \frac{e^{-x}}{2} + \frac{e^x}{2} - \frac{e^{-x}}{2} = 1$$

$$\frac{2e^x}{2} = 1$$

$\ln 1 = 0$

$\ln e^x = \ln 1$

2) $\int \frac{2x}{2x+5} dx =$

a) $x - \ln|2x+5| + C$.

b) $\ln|2x+5| + C$.

c) $x + \frac{5}{2}\ln|2x+5| + C$

d) $x - \frac{5}{2}\ln|2x+5| + C$

$$\int \frac{2x}{2x+5} \qquad \int \frac{2x}{2x(1+\frac{5}{2x})} \qquad \frac{1}{1+\frac{5}{2x}}$$

3) The Series $\sum_{0}^{\infty} \frac{1}{(n+1)(n+2)}$

a) Converges to 1

b) Converges to 0

c) Converges to $\frac{1}{2}$

d) Diverges

$$\frac{1}{n^2+3n+2} = \int_0^{\infty} \frac{A}{n+1} + \frac{B}{n+2}$$

$$A(n+2) + B(n+1) = 1$$

$-B = 1 \qquad B = -1$

$A(1) + 0 = 1 \qquad A = 1$

$$\frac{1}{n+1} - \frac{1}{n+2}$$

$$= \ln|n+1| - \ln|n+2|$$

$$\ln\left|\frac{n+1}{n+2}\right|_0^{\infty}$$

$$\lim \ln\left|\frac{n+1}{n+2}\right|$$

$\ln \infty \quad \ln|1| - \ln|\frac{1}{2}|$

$0 - 1$

4) $\sinh(5 \ln x) =$

a) $\frac{5}{2}(x - \frac{1}{x})$

b) $\frac{1}{2}(x^5 - \frac{1}{x^5})$

c) $\frac{1}{2}(x^5 + \frac{1}{x^5})$

d) $2x$.

$$\frac{e^x - e^{-x}}{2} = \frac{e^{5\ln x} - e^{-5\ln x}}{2} = \frac{e^{\ln x^5} - e^{\ln x^{-5}}}{2}$$

$$\frac{x^5 - \frac{1}{x^5}}{2} = \frac{1}{2}\left(x^5 - \frac{1}{x^5}\right)$$

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5) Consider $\sum_{1}^{\infty} a_n$ Where $a_n \geq 0$ Then

a) If $\lim_{x \to \infty} a_n = 0$ then $\sum_{1}^{\infty} a_n$ converges.

b) If $\lim_{x \to \infty} a_n \neq 0$ then $\sum_{1}^{\infty} a_n$ diverges

c) If $\sum_{1}^{\infty} a_n$ diverges then $\lim_{x \to \infty} a_n \neq 0$

d) If $\sum_{1}^{\infty} a_n$ converges then $\lim_{x \to \infty} a_n \neq 0$

$\left(1-\frac{1}{n}\right)^n$ $\quad 1 \cdot \frac{x}{1}$

$\left(1-\frac{1}{\frac{1}{x}}\right)^{\frac{1}{x}}$

$(1-x)^{\frac{1}{x}}$ $\quad n = \frac{1}{x}$

$x \cdot \frac{x}{1} = x^2$

$\left(1+\frac{1}{n}\right)^n \left(1+\frac{x}{n}\right)^n$

$e \left(1+\frac{x}{\frac{1}{x}}\right)^{\frac{1}{x}} = (1+x^2)^{\frac{1}{x}}$

$= e^{x}$

6) The sequence $(a_n) = (1-\frac{1}{n^2})^n$

a) Converges to 1

b) Converges to e

c) Converges to e^{-1}

d) diverges

$\left(1+\frac{x}{n}\right)^{n}$ $\quad \left(1-\frac{1}{n^2}\right)^n$ $\quad \frac{1}{\frac{1}{x^2}}$ $\quad 1 \cdot \frac{x^2}{1}$

$n = \frac{1}{x}$ $\quad 1+$

$\left(1-\frac{1}{(\frac{1}{x})^2}\right)^{\frac{1}{x}} = (1-x^2)^{\frac{1}{x}}$ $\quad \left(1+\frac{\frac{1}{n}}{n}\right)^{\frac{1}{x}}$

$\frac{1}{n} = x$

7) $\int \tan^{-1} x \, dx =$

a) $\ln(\cos^{-1} x) + c$

b) $x\tan^{-1} x - \ln(x^2+1) + c$

c) $x\tan^{-1} x - \ln\sqrt{x^2+1} + c$

d) $x\tan^{-1} x - \dfrac{x}{x^2+1} + c$

$u = \tan^{-1}(x)$ $\quad dv = dx$

$du = \frac{1}{1+x^2}$ $\quad v = x$

$u = x^2$

$du = 2x\,dx$

$x\tan^{-1}(x) - \int \frac{x}{1+x^2}$

$x\tan^{-1}(x) - \frac{1}{2}\int \frac{du}{1+u}$

$= x\tan^{-1}(x) - \frac{1}{2}\ln|1+u| + c$

$\boxed{x\tan^{-1}(x) - \frac{1}{2}\ln(x^2+1)^{\frac{1}{2}} + c}$

$x\tan^{-1}(x) - \ln\sqrt{x^2+1} + c$

8) $\sum_{1}^{\infty} (\ln(x))^n$ Converges If

a) $-1 < x < 1$

b) $0 < x < e$

c) $0 < x < 1$

d) $e^{-1} < x < e$

$x^n = 0$ $\quad |x| < 1$

$-1 < x < 1$

$\ln e^{-1}$ $\quad \ln e^{1}$

$e^{2t} = 2e^{2t}$ $\int \cancel{3} e^{t}\left(\frac{e^{t}+e^{-t}}{\cancel{2}}\right)$

$\int 3e^{2t} + 3(1)$

9) $\int_{\ln 2}^{\ln 7} 6e^{t}\cosh t\, dt =$

$\int 3e^{2t} + 3$ $\quad \int 3(e^{2t}+1)$

a) $\frac{135}{2}$

$3\int e^{2t}+1 = 3\left[\frac{e^{2t}}{2}+t\right]_{\ln 2}^{\ln 7}$

b) $\frac{135}{2} + 3\ln 7 - 3\ln 2$

c) $3\ln\frac{7}{3}$

$3\left[\frac{e^{\ln 7^2}}{2} + \ln 7 - \left(\frac{e^{\ln 2^2}}{2} + \ln 2\right)\right]$

d) 1.

$3\left[\frac{49}{2} + \ln 7 - \frac{4}{2} - \ln 2\right]$

$3\left[\frac{45}{2} + \ln 7 - \ln 2\right]$

10) The Series $\sum_{1}^{\infty}\frac{2^n-2}{3^n}$

$\boxed{\frac{135}{2} + 3\ln 7 - 3\ln 2}$

a) Converges to 1

b) Converges to $\frac{3}{4}$

$\sum \frac{2^n}{3^n} - \sum \frac{2}{3^n}$

c) Converges to 2

d) Converges to $\frac{3}{2}$

$\sum^{\infty} \left(\frac{2}{3}\right)^n - 2\sum_{1} \left(\frac{1}{3}\right)^n$

$\frac{2}{3} + \frac{4}{9}$ $\quad 2\left(\frac{1}{3}\right)^1 + 2\left(\frac{1}{9}\right)$ $\quad \frac{2}{3}$ $\quad \frac{2}{9}$ $\quad \frac{1}{3}$

$\frac{a}{1-r}$ $\quad \frac{\frac{2}{3}}{1-\frac{2}{3}} - \frac{\frac{2}{3}}{1-\frac{1}{3}}$ $\quad \frac{\frac{2}{3}}{\frac{1}{3}}$ $\quad 2 - 1 = 1$

Problem 2. (30 pts) Compute

a) $\int \frac{x^2}{\sqrt{4-x^2}}dx$ $\quad a^2 - x^2 \Rightarrow (2)^2 - x^2$

$\Rightarrow \boxed{x = 2\sin\theta} \Rightarrow \sin\theta = \frac{x}{2}$

$dx = 2\cos\theta\, d\theta$

$\boxed{1-\sin^2\theta = \cos^2\theta}$

$\int \frac{4\sin^2\theta\,[2\cos\theta\, d\theta]}{\sqrt{4-4\sin^2\theta}} = \int \frac{4\sin^2\theta\,[2\cos\theta\, d\theta]}{\sqrt{4(1-\sin^2 x)}}$

$= \int \frac{4\sin^2\theta\,[2\cos\theta\, d\theta]}{\sqrt{4\cos^2\theta}} = \int \frac{4\sin^2\theta\,[\cancel{2\cos\theta}\, d\theta]}{\cancel{2\cos\theta}}$

$= \int 4\sin^2\theta \Rightarrow \boxed{\sin^2\theta = \frac{1-\cos 2\theta}{2}}$

$= 4\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta = 2\int 1 - \cos 2\theta\, d\theta$

$= 2\left[\theta - \frac{\sin 2\theta}{2} + c\right] = \boxed{2\theta - \sin 2\theta + C}$

$\theta = \sin^{-1}(x)$ $\quad \sin 2\theta = x$ $\Rightarrow \boxed{I = 2\sin^{-1}(x) - x + C}$

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b) $\int \sin^3 x \cos^2 x\, dx$ $= \int \sin^2 x \cos^2 x \cdot \sin x$

$= \int (1-\cos^2 x)(\cos^2 x) \cdot \sin x$

$u = \cos x \Rightarrow du = -\sin x\, dx$

$I = -\int (1-u^2)\, u^2\, du = -\int u^2 - u^4\, du$

$= -\left[\frac{u^3}{3} - \frac{u^5}{5}\right] = \frac{-u^3}{3} + \frac{u^5}{5} + c = \boxed{-\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C}$

c) $\int \frac{y^2+2y+1}{(y^2+1)^2}\, dy$. ~~$= \int \frac{(y+1)^2}{y^2}$~~ $= \frac{Ay+B}{(y^2+1)} + \frac{Cy+D}{(y^2+1)^2}$

$y^2+2y+1 = Ay+B + (Cy+D)(y^2+1)$

$y^2+2y+1 = Ay + B + [Cy^3 + Cy + Dy^2 + D]$

$y^2+2y+1 = (A+C)y + Dy^2 + Cy^3 + (B+D)$

$(A+C)y = 2y \Rightarrow \boxed{A+C=2}$ $\quad Cy^3 = 0 \Rightarrow C = 0$

$Dy^2 = y^2 \Rightarrow \boxed{D=1}$ $\quad (B+D) = 1$

$B + 1 = 1 \Rightarrow \boxed{B=0}$

$A + C = 2 \Rightarrow A + 0 = 2 \Rightarrow \boxed{A=2}$

$\boxed{A=2}$ $\boxed{B=0}$ $\boxed{C=0}$ $\boxed{D=1}$

$I: \int \frac{2y}{y^2+1} + \frac{1}{(y^2+1)^2} = \boxed{\ln|y^2+1| + \int \frac{1}{(y^2+1)^2} + C}$

~~$= \int (y^2+1)^{-2}$~~ ~~$u = y^2 \Rightarrow du = 2y$~~

$=$

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Problem 3. (30 pts) Determine which of the following integral (or series) converges and which diverges.

a) $\int_{-1}^{1} \frac{dx}{x^{\frac{2}{3}}}$.

$= \cancel{\int x^{-\frac{2}{3}}}$ $\lim_{n\to\infty} \int_n^1 \cancel{x^{-\frac{2}{3}}} = \lim_{n\to\infty} \int_n^1 \frac{x^{\frac{1}{3}}}{\frac{1}{3}}$

$= 3 \lim_{n\to\infty} \sqrt[3]{x} = 3(\infty) = \infty$

$\boxed{\text{diverge}}$

b) $\int_{1}^{\infty} \frac{dx}{\sqrt{e^x - x}}$

$\Rightarrow \boxed{\frac{1}{\sqrt{ex}}}$ ~~Compare~~ Compare

$\frac{\sqrt{ex}}{\sqrt{e^x - x}} = \sqrt{\frac{ex}{ex - x}} = \sqrt{1} = 1$

How

$\boxed{\text{Converge}}$

c) $\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^{100}}$

$\cancel{\frac{1}{n \ln n}}$

$\frac{1}{n}$ Compare ~~Converge~~

~~$n > \ln$~~ $n > 1$

diverge

P-series $\boxed{P = 1}$